a. Write $z=-2+3i$ in polar form

b. Write $3e^{i\frac{\pi}{6}}$ in rectangular coordinates.

c. Draw and label the triangle relating rect. to polar coordinates

d. Compute $\frac{1}{-2+3i}$ in polar form

e. Find the cube root of 1

23/3/25

a. $z=-2+3i$

$r=\sqrt{(-2)^2+3^2}$

$=\sqrt{13}$

$\theta=\tan^{-1}\frac{3}{-2}$

$z=\sqrt{13}\,e^{i\tan^{-1}\frac{3}{-2}}$

b. $3e^{i\frac{\pi}{6}}=3\left(\cos\frac{\pi}{6}+i\sin\frac{\pi}{6}\right)$

$=3\left(\frac{\sqrt{3}}{2}+i\frac{1}{2}\right)$

$=\frac{3\sqrt{3}}{2}+\frac{3}{2}i$

c.

d. $\frac{1}{-2+3i}$

$=\frac{-2-3i}{4-9i^2}$

$=-\frac{2}{13}-\frac{3}{13}i$

$r=\sqrt{\left(-\frac{2}{13}\right)^2+\left(-\frac{3}{13}\right)^2}$

$=\sqrt{\frac{4+9}{169}}$

$=\frac{\sqrt{13}}{13}$

$\theta=\tan^{-1}\frac{-\frac{3}{13}}{-\frac{2}{13}}$

$=\tan^{-1}\frac{-3}{-2}$

$z=\frac{\sqrt{13}}{13}\,e^{i\tan^{-1}\frac{-3}{-2}}$

e. $\sqrt[3]{1}$

$z^3 = 1$

$r^3 e^{in\theta} = 1 \cdot e^{2k\pi i}$, $k = 0, \pm 1, \pm 2, \ldots$

$r^3 = 1$, $n\theta = 2k\pi$

$r = 1$ $\theta = \frac{2k\pi}{n}$, $k = 0, 1, \ldots, n-1$

$\theta' = \theta + 2a\pi$, $a \in \mathbb{Z}$

$$
\begin{aligned}
e^{i\theta'} &= e^{i(\theta + 2a\pi)} \\
&= e^{i\theta} \cdot e^{i2a\pi} \\
&= e^{i\theta} \cdot (e^{i\pi})^a \\
&= e^{i\theta}
\end{aligned}
$$

$e^{\frac{2k\pi i}{3}}$, $k = 0, 1, 2$

$$
\begin{aligned}
z &= e^0, e^{\frac{2\pi}{3}i}, e^{\frac{4\pi}{3}i} \\
&= 1, \cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3}, \cos\frac{4\pi}{3} + i\sin\frac{4\pi}{3} \\
&= 1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i
\end{aligned}
$$